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DESIGN AND ANALYSIS OF A LAMINATED COMPOSITE TUBE

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ABSTRACT: This report presents the detailed process for the design and analysis of a laminated composite tube those subjects to the internal pressure as well as axial compression. The software package MATLAB was employed to determine the optimum winding angles that enable the tube to withstand the axial compression and internal pressure without mechanical failure and obtain a maximum angle of twist under these loading conditions. Based on the maximum stress failure criteria, the results show that the optimum winding angles are between $\alpha = -75^o$ and $\beta = -39^o$ that enables to achieve the maximum angle of twist of 12.99^o.

KEYWORDS: *Laminated composite tube, Classical Laminate Theory, Maximum stress failure criteria, Filament winding*

I. INTRODUCTION

Laminated composites have been extensively used in several engineering industries, such as civil infrastructure, automotive and aerospace. Generally, composites can be broadly defined as a multi-phase material, which includes fibre and matrix [1]. Combining different kinds of materials, it is possible to achieve better overall properties in terms of high strength and stiffness-to-weight ratios than each of original constituents [2]. Consequently, conducting a failure analysis towards composites is essential. There are substantial numbers of publications available on the delamination or buckling in the laminated composite tube using finite element methods, but limited studies are conducted using analytical methods. This paper deals with the analytical investigation of matrix tensile failure, matrix compression failure, fibre tensile failure and fibre compression failure using maximum stress failure criteria.

The main purpose of this study is to design a cylindrical tube of 4 layers of composites with a layup of [α/β/α/β], and the winding tapes wrapped onto a $\varphi 50mm$ mandrel was cut from a union direction carbon/epoxy prepreg sheet. Taken the practical consideration into account, these two winding angles should be ranged between [-75^o, -30^o] or [30^o, 75^o]. The length of the tube is *300mm* and the thickness of the prepreg is 0.25 mm.The composites tube is required to sustain the internal pressure 3 MPa and axial compression of 25N. A combination of α and β is required to identify so as to achieve the maximum twist angle under stated loading conditions.

II. ANALYTICAL MODEL

2.1 Boundary and loading conditions

A schematic illustration of the boundary conditions and loading conditions has been shown in figure 2. As it suggested, the cylindrical tube was fixed at one end. An axial compressive load (F) and twist under this compression are applied to the opposite ends. The internal pressure is also uniformly distributed on the inner surface of the tube.

Table 1. Material properties of carbon-epoxy

Material constants	Strengths
$E_1 = 236GPa$	$\sigma_{1t}^* = Xt = 3800MPa$
$E_2 = 5GPa$	$\sigma_{1c}^* = Xc = 689MPa$
$G_{12} = 2.6GPa$	$\sigma_{2t}^* = Yt = 41MPa$
$\upsilon_{12} = 0.25$	$\sigma_{2c}^* = Yc = 107MPa$
	$\tau_{12}^* = S = 69MPa$

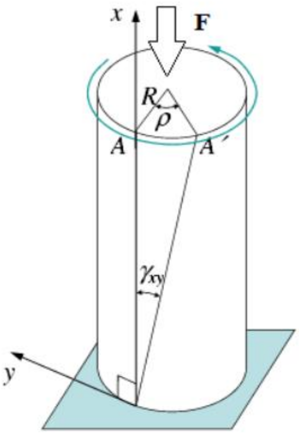


Figure 2. Boundary and loading conditions

2.2 Material properties

A composite is a multi-phase material that made from two or more than two constituents with different material properties. Generally, the constituents of composites can be divided into two groups. One phase is continuous named matrix, while the other phase is discontinuous called the reinforcement, such as fibrous or particulates. The material used in this design is carbon-epoxy and the materials properties have been presented in table 1.

III. METHODOLOGY

IV.

3.1 Analysis of orthotropic lamina

The elasticity mechanics suggests that orthotropic materials exist two reflectional symmetries. One is about 1-plane and the other is about 2-plane (1-2-3 is material principle axes). This type of materials has 9 independent that are Young's moduli E_1, E_2 and E_3, Poisson's ratios v_{12}, v_{13} and v_{23} and shear moduli G_{12}, G_{13} and G_{23} [3]. When conducting an analysis for orthotropic composites, a thin lamina can be regarded as the basic building unit, which is assumed to subject to in-plane loading; therefore, $\sigma_3 = \tau_{23} = \tau_{13} = 0$ and the strain-stress relationship can be simplified as:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\upsilon_{21}}{E_2} & 0 \\ -\frac{\upsilon_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}, \text{or} \{\varepsilon\} = [R]\{\sigma\} \quad (1)$$

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}\frac{E_1}{1-v_{12}v_{21}} & \frac{v_{12}E_2}{1-v_{12}v_{21}} & 0\\ \frac{v_{12}E_2}{1-v_{12}v_{21}} & \frac{E_2}{1-v_{12}v_{21}} & 0\\ 0 & 0 & G_{12}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}, \text{or} \{\sigma\}=[Q]\{\varepsilon\} \quad (2)$$

3.2 Coordinate Transformation

In the current study, the loading condition is given in structure coordinates system (*x-y-z*), while the failure analysis is performing in the material principle axes (*1-2-3*), as shown in figure 3. To do so, the coordinate transformation is required and the coordinate transformation matrix under plane stress state is defined as:

$$[\mathrm{T}]=\begin{bmatrix}\cos^2\theta & \sin^2\theta & -2\cos\theta\sin\theta\\ \sin^2\theta & \cos^2\theta & 2\cos\theta\sin\theta\\ \cos\theta\sin\theta & -\cos\theta\sin\theta & \cos^2\theta-\sin^2\theta\end{bmatrix} \quad (3)$$

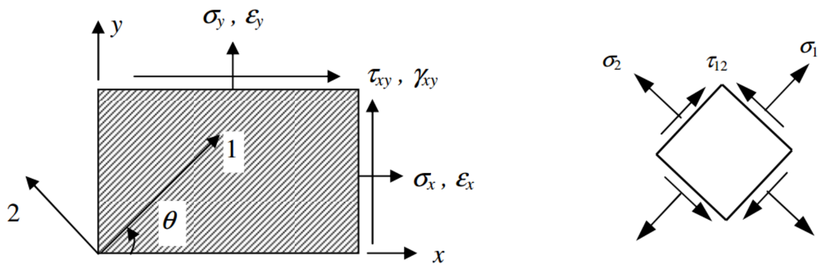


Figure 3. Coordination transformation between *x-y-z* and *1-2-3*

The coordination transformation between structure coordinates system (*x-y-z*) and material principle axes (*1-2-3*) are as follows [4]:

$$\begin{array}{ll}\{\sigma'\}=[T]\{\sigma\} & \{\sigma\}=[T]^{-1}\{\sigma'\}\\ \{\varepsilon'\}=[T]^{-T}\{\varepsilon\} & \{\varepsilon\}=[T]^{T}\{\varepsilon'\}\\ [Q']=[T][Q][T]^{T} & [Q]=[T]^{-1}[Q'][T]^{-T}\\ [R']=[T]^{-T}[R][T]^{-1} & [R]=[T]^{T}[R'][T]\end{array} \quad (4)$$

3.3 Classic Laminate Theory (CLT)

CLT, using a two-dimensional presentation of a three-dimensional problem, is a direct extension of classical beam theory. Due to the presence of stress jump between each ply, the classical beam theory is invalid for the overall composites structure, but it should be acknowledged that the beam theory is still valid in each ply. According to the Love-Kirchhoff hypothesis, it assumes that the normal to the laminate remains normal, straight and up stretched; therefore, the kinematic equations can be obtained by neglecting the out-of-plane strain. The three cornerstones equations of CLT that will be used in this report are Kinematics, Constitutive and force resultant equations and are summarized as follows:

Kinematics equation:

$$\{\varepsilon'\}=\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\{\varepsilon^0\}+z\{k\} \quad (5)$$

Constitutive equation:

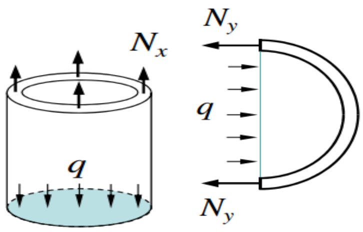
$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}Q'_{11} & Q'_{12} & Q'_{16}\\ Q'_{21} & Q'_{22} & Q'_{26}\\ Q'_{61} & Q'_{62} & Q'_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix},\ \text{or}\ \{\sigma'\}=[Q']\{\varepsilon'\} \tag{6}$$

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{bmatrix}R'_{11} & R'_{12} & R'_{16}\\ R'_{21} & R'_{22} & R'_{26}\\ R'_{61} & R'_{62} & R'_{66}\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix},\ \text{or}\{\varepsilon'\}=[R']\{\sigma'\} \tag{7}$$

Force resultants:

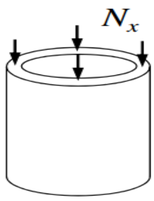
$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\int_{z^{bottom}}^{z^{top}}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}dz,\quad \begin{Bmatrix}M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\int_{z^{bottom}}^{z^{top}}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}z\,dz \tag{8}$$

Figure 4 shows the generalized stress under internal pressure and axial compression that can be calculated by equations (9) and (10), respectively.



$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{1}{2}qR\\ qR\\ 0\end{Bmatrix} \tag{9}$$

(a)



$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{P}{2\pi R}\\ 0\\ 0\end{Bmatrix} \tag{10}$$

(b) Figure 4. Generalised stresses under internal pressure (a) and axial compression (b)

Substituted the stress-strain relationship (6) under structure coordinate system into membrane force and bending moments in equation (8). The generalised stress-strain relationship can be obtained as follow:

$$\begin{Bmatrix}N\\ M\end{Bmatrix}=\begin{bmatrix}A & B\\ B & D\end{bmatrix}\begin{Bmatrix}\varepsilon^0\\ k\end{Bmatrix} \tag{11}$$

Where,

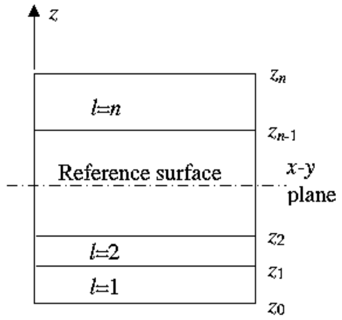


Figure 5. Illustration of composite beam structure

$$[A] = \int_{z_{bottom}}^{z_{top}} [Q'] \, dz = \sum_{l=1}^{n} [Q']_l (z_l - z_{l-1}) \tag{12}$$

$$[B] = \int_{z_{bottom}}^{z_{top}} [Q'] \, z dz = \frac{1}{2} \sum_{l=1}^{n} [Q']_l ({z_l}^2 - {z_{l-1}}^2) \tag{13}$$

$$[D] = \int_{z_{bottom}}^{z_{top}} [Q'] \, z^2 dz = \frac{1}{3} \sum_{l=1}^{n} [Q']_l ({z_l}^3 - {z_{l-1}}^3) \tag{14}$$

3.4 Maximum stress failure criterion

In the two-dimensional case, based on the maximum stress failure criterion, the material fails when any of the following conditions is violated for a given stress state in a composite. This criterion needs five independent strength properties, where σ_{1t}^* , σ_{1c}^* , σ_{2t}^* , σ_{2c}^* and τ_{12}^* are tensile, compressive and shear strength in material principal axes [5].

$$\frac{\sigma_1}{\sigma_{1t}^*} \le 1 \text{if } \sigma_1 \ge 0 , \frac{|\sigma_1|}{\sigma_{1c}^*} \le 1 \quad \text{if } \sigma_1 \le 0 ; \frac{\sigma_2}{\sigma_{2t}^*} \le 1 \quad \text{if } \sigma_2 \ge 0 , \frac{|\sigma_2|}{\sigma_{2c}^*} \le 1 \quad \text{if } \sigma_2 \le 0 ; \frac{|\tau_{12}|}{\tau_{12}^*} \le 1 \tag{15}$$

3.5 Analytical design example

Step 1: Several data was input into the MATLAB. These data has been shown in table 2 and table 3.

Table 2. Input data of material properties

Input data:			
$q = 3 \times 10^6$	$P = -25000$	$R = 0.0255$	$L = 0.300$
$E_1 = 236 \times 10^6$	$E_2 = 5 \times 10^9$	$G = 2.6 \times 10^9$	$v = 0.25$

where q is internal pressure, P is axial compression, R is the radius of tube, L is the length of the tube, E_1 is the Young's moduli in fibre direction, E_2 is the Young's moduli in transverse direction, G is the shear moduli, v is the Poison's ratio.

Table 3. Input data of material strength

Strengths:				
$X_t = 38 \times 10^8$	$Y_t = 41 \times 10^6$	$X_c = 689 \times 10^6$	$Y_c = 107 \times 10^6$	$S = 69 \times 10^6$

where X_t and X_c are tensile and compressive strength in fibre direction, Y_t and Y_c are tensile and compressive strength in transverse direction and S is the shear strength. An example of design analysis has been done by assuming that composite tube is designed with a layup of [60/45/60/45] and a uniform thickness of 0.25mm.

Step 2: Substitute material properties into equation (2) and calculate local stiffness matrix [Q] in material principle axes:

$$[Q] = \begin{bmatrix} \frac{E_1}{1 - v_{12}v_{21}} & \frac{v_{12}E_2}{1 - v_{12}v_{21}} & 0 \\ \frac{v_{12}E_2}{1 - v_{12}v_{21}} & \frac{E_2}{1 - v_{12}v_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} = \begin{bmatrix} 236.31 & 1.25 & 0 \\ 1.25 & 5.01 & 0 \\ 0 & 0 & 2.6 \end{bmatrix} GPa$$

Step 3: Substitute winding angle α and β into equation (3) and calculate the coordinate transformation matrix [T (60)] and [T (45)].

$$[\mathrm{T}(60)] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & 2\cos\theta\sin\theta \\ \cos\theta\sin\theta & -\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} = \begin{bmatrix} 0.25 & 0.75 & -0.87 \\ 0.75 & 0.25 & 0.87 \\ 0.43 & -0.43 & -0.50 \end{bmatrix}$$

$$[\mathrm{T}(45)] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & 2\cos\theta\sin\theta \\ \cos\theta\sin\theta & -\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & -1 \\ 0.5 & 0.5 & 1 \\ 0.5 & -0.5 & 0 \end{bmatrix}$$

Step 4: Substitute [T (60)] and [T (45)] into $[Q'] = [T][Q][T]^T$ that has been shown in equation (4).

$$[Q']_{+60^0} = [T(60)][Q][T(60)]^T = \begin{bmatrix} 20.01 & 44.08 & 25.35 \\ 44.08 & 135.66 & 74.81 \\ 25.35 & 74.81 & 45.43 \end{bmatrix} GPa$$

$$[Q']_{+45^0} = [T(45)][Q][T(45)]^T = \begin{bmatrix} 63.56 & 58.36 & 57.83 \\ 58.36 & 63.56 & 57.83 \\ 57.83 & 57.83 & 59.70 \end{bmatrix} GPa$$

Step 5: Substitute $[Q']_{+60^0}$ and $[Q']_{+45^0}$ into equation (12) to calculate the stiffness matrix [A].

$$[A]=\int_{z^{bottom}}^{z^{top}}[Q']\,dz=2\times z[Q']_{+45^0}\Big|_{z_0}^{z_1}+2\times z[Q']_{-45^0}\Big|_{z_1}^{z_2}=\begin{bmatrix}41.78 & 51.22 & 41.59\\ 51.22 & 99.61 & 66.32\\ 41.59 & 66.32 & 52.57\end{bmatrix}\times 10^6 N/m$$

Because, the value of bending moment in this case is relatively small; therefore, the bending moment has been ignored. Then the curvature $\{k\}$ can be assumed to be zero. Once $\{k\}=0$, there is no need to calculate the [B] matrix.

Step 6: According to the load case 1 (internal pressure), the generalised stresses can be calculated using equation (9).

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{1}{2}qR\\ qR\\ 0\end{Bmatrix}=\begin{Bmatrix}3.825\\ 7.65\\ 0\end{Bmatrix}\times 10^4 N/m$$

Then, using the equation (11) as mentioned before, the membrane strain can be obtained. Recall that $\{k\}=0$.

$$\{\varepsilon^0\}=[A]^{-1}\{N\}=\begin{Bmatrix}0.0050\\ 0.0052\\ -0.0105\end{Bmatrix}$$

Using Kinematics equation (5), $\{\varepsilon'\}=\{\varepsilon^0\}+z\{k\}$, the strain under structure coordinates system(*x*-*y*-*z*) can be obtained.

$$\{\varepsilon'\}=\{\varepsilon^0\}=\begin{Bmatrix}0.0050\\ 0.0052\\ -0.0105\end{Bmatrix}$$

Step 7: Using the coordinate transformation in equation (3), the stress under material principle axes (1-2-3) can be calculated by $\{\varepsilon\}=[T]^T\{\varepsilon'\}$, and then the stress under material principle axes (1-2-3) can be calculated based on the stress-strain relationship in equation (4).

Lamina 1:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}=[T(60^0)]^T\{\varepsilon'\}=\begin{bmatrix}0.25 & 0.75 & 0.43\\ 0.75 & 0.25 & -0.43\\ -0.87 & 0.87 & -0.50\end{bmatrix}\begin{Bmatrix}0.0050\\ 0.0052\\ -0.0105\end{Bmatrix}=\begin{Bmatrix}0.0006\\ 0.0096\\ 0.0054\end{Bmatrix},$$

$$\{\sigma\}=\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix}=[Q]\{\varepsilon\}=\begin{bmatrix}236.31 & 1.25 & 0\\ 1.25 & 5.01 & 0\\ 0 & 0 & 2.6\end{bmatrix}\begin{Bmatrix}0.0006\\ 0.0096\\ 0.0054\end{Bmatrix}GPa=\begin{Bmatrix}151.79\\ 49.02\\ 14.05\end{Bmatrix}MPa$$

Lamina 2:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}=[T(45^0)]^T\{\varepsilon'\}=\begin{bmatrix}0.50 & 0.50 & 0.50\\ 0.50 & 0.50 & -0.50\\ -1 & 1 & 0\end{bmatrix}\begin{Bmatrix}0.0050\\ 0.0052\\ -0.0105\end{Bmatrix}=\begin{Bmatrix}-0.0002\\ 0.0104\\ 0.0002\end{Bmatrix}$$

$$\{\sigma\} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [Q]\{\varepsilon\} = \begin{bmatrix} 236.31 & 1.25 & 0 \\ 1.25 & 5.01 & 0 \\ 0 & 0 & 2.6 \end{bmatrix} \begin{Bmatrix} -0.0002 \\ 0.0104 \\ 0.0002 \end{Bmatrix} GPa = \begin{Bmatrix} -23.13 \\ 51.82 \\ 0.395 \end{Bmatrix} MPa$$

Step 8: Using the maximum stress failure criterion to check failure in fibre direction and transverse direction.

Lamina 1: $\frac{\sigma_1}{\sigma_{1t}^*} = \frac{151.79}{3800} = 0.04 < 1,\quad \frac{\sigma_2}{\sigma_{2t}^*} = \frac{49.02}{41} = 1.20 > 1,\quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{14.05}{69} = 0.203 < 1$

Lamina 2: $\frac{|\sigma_1|}{\sigma_{1c}^*} = \frac{23.13}{689} = 0.034 < 1,\quad \frac{\sigma_2}{\sigma_{2t}^*} = \frac{51.82}{41} = 1.26 > 1,\quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{0.395}{69} = 5.73 \times 10^{-3} < 1$

Therefore, it can be concluded that transverse tensile failure is observed in both laminate 1 and laminate 2.

Step 9: Repeat the steps from 6 to 8, and calculated the stress under material principle axes (1-2-3), which can be used for the failure analysis. For the sake of brevity, only the results under axial compression are presented.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{P}{2\pi R} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -15.60 \\ 0 \\ 0 \end{Bmatrix} \times 10^4 N/m, \quad \{\varepsilon'\} = \{\varepsilon^0\} = \begin{Bmatrix} -0.0178 \\ -0.0014 \\ 0.0158 \end{Bmatrix}$$

$$\text{Angle of twist} = \frac{\gamma_{xy} L}{R} = \frac{0.0158 \times 0.3}{0.0255} \times \frac{180}{\pi} = 10.667^0$$

Table 4. Strain and stress under material principle axes (internal pressure)

Lamina 1		**Lamina 2**	
Strain	Stress	Strain	Stress
$\{\varepsilon\} = \begin{Bmatrix} 0.0014 \\ -0.0205 \\ 0.0063 \end{Bmatrix}$	$\{\sigma\} = \begin{Bmatrix} 296.14 \\ -101.10 \\ 16.32 \end{Bmatrix} MPa$	$\{\varepsilon\} = \begin{Bmatrix} -0.0017 \\ -0.0175 \\ 0.0164 \end{Bmatrix}$	$\{\sigma\} = \begin{Bmatrix} -417.41 \\ -89.70 \\ 42.59 \end{Bmatrix} MPa$

Failure analysis

Lamina 1: $\frac{\sigma_1}{\sigma_{1t}^*} = \frac{296.14}{3800} = 0.078 < 1,\quad \frac{|\sigma_2|}{\sigma_{2c}^*} = \frac{101.10}{107} = 0.945 < 1,\quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{16.32}{69} = 0.236 < 1$

Lamina 2: $\frac{|\sigma_1|}{\sigma_{1c}^*} = \frac{417.41}{689} = 0.606 < 1,\quad \frac{|\sigma_2|}{\sigma_{2c}^*} = \frac{89.70}{107} = 0.838 < 1,\quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{42.59}{69} = 0.617 < 1$

All failure induces are less than 1, so no failure is observed under internal pressure.

IV. RESULTS AND DISCUSSION

The example of design procedure has been shown in the above section 3.5. In order to find the optimum winding angle that sustains the internal pressure and axial compression, the software package 'MATLAB' was employed. Two loops were created to calculate all the combinations of α and β and find one set of α and β that maximize the angle of twist. The value of both α and β increase from -75^o to -30^o and the increment is 1^o. Once α increased by 1^o, the program keeps the value of α constant and calculates all the combinations of it with β until β increasing from -75^o to -30^o. After all the calculation has been down, it can be found that the minimum angle of twist achieved when $\alpha = -75^o$and $\beta = -39^o$. Figure 6 shows the 3-D plot of all the combination between α and β and twist angle.

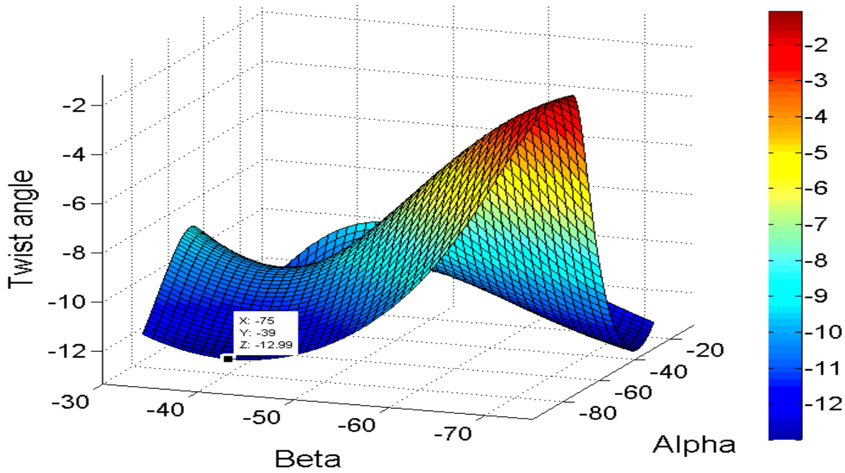


Figure 6.Twist angle with all combination of α and β

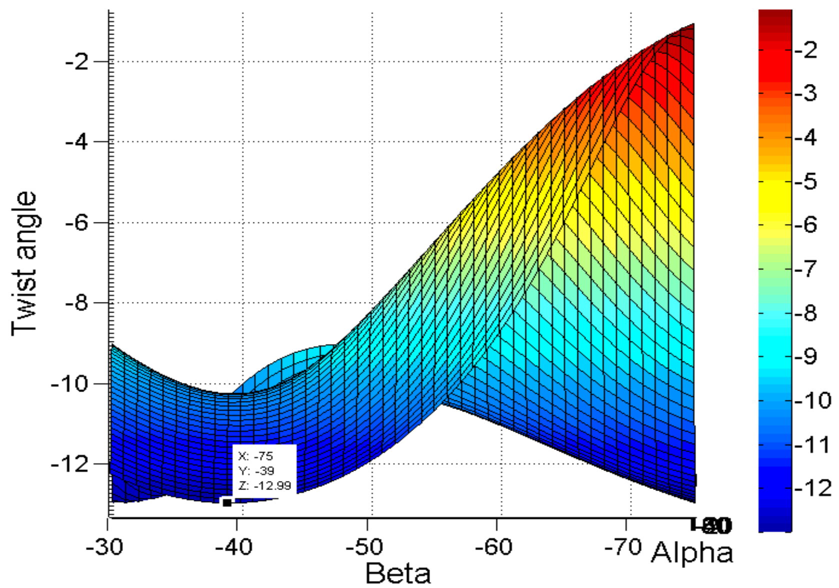


Figure 7. Orthographic projection with a projection plane perpendicular to α axes

Figure 7 shows the orthographic projection of figure 6 and the optimum combination of α and β has been plotted. The twist angle of -12.99 is the global minimum value. Due to the direction of twist is in the anti-clockwise direction. According to the right-hand rule, the positive angle of twist is due to tensile, but in this case, the tube subjected to a compression; therefore, the maximum angle of twist is the minimum value in figure 7.

Once the maximum angle of twist has been identified, the maximum stress is applied to check whether the material is safe when it is subjected to the loading condition. Substituted $\alpha = -75^o$and $\beta = -39^o$ into step 3 in section 3.5 and the results of stress for laminate 1 and laminate 2 in material principle axes are as follows:

Table 5. Stress under material principle axes

Internal pressure		**Axial compression**	
Lamina 1	Lamina 2	Lamina 1	Lamina 2
$\{\sigma\} = \begin{Bmatrix} 130.9 \\ 37.01 \\ -15.26 \end{Bmatrix} MPa$	$\{\sigma\} = \begin{Bmatrix} 22.83 \\ 38.73 \\ 11.97 \end{Bmatrix} MPa$	$\{\sigma\} = \begin{Bmatrix} 156.7 \\ -95.8 \\ 27.68 \end{Bmatrix} MPa$	$\{\sigma\} = \begin{Bmatrix} -284.3 \\ -88.73 \\ -41.11 \end{Bmatrix} MPa$

Failure analysis for internal pressure condition:

Lamina 1: $\frac{\sigma_1}{\sigma_{1t}^*} = \frac{130.9}{3800} = 0.035 < 1, \quad \frac{\sigma_2}{\sigma_{2t}^*} = \frac{37.01}{41} = 0.903 < 1, \quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{15.26}{69} = 0.221 < 1$

Lamina 2: $\frac{\sigma_1}{\sigma_{1t}^*} = \frac{22.83}{3800} = 0.006 < 1, \quad \frac{\sigma_2}{\sigma_{2t}^*} = \frac{38.73}{41} = 0.945 < 1, \quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{11.97}{69} = 0.173 < 1$

Failure analysis for axial compression condition:

Lamina 1: $\frac{\sigma_1}{\sigma_{1t}^*} = \frac{156.7}{3800} = 0.041 < 1, \quad \frac{|\sigma_2|}{\sigma_{2c}^*} = \frac{95.8}{107} = 0.895 < 1, \quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{27.68}{69} = 0.401 < 1$

Lamina 2: $\frac{|\sigma_1|}{\sigma_{1c}^*} = \frac{284.3}{689} = 0.413 < 1, \quad \frac{|\sigma_2|}{\sigma_{2c}^*} = \frac{88.73}{107} = 0.829 < 1, \quad \frac{|\tau_{12}|}{\tau_{12}^*} = \frac{41.11}{69} = 0.596 < 1$

All the values are less than 1, therefore no failure is predicted in plies 1 and 2. Based on the calculation by employing MATLAB and failure analysis, it can be concluded that $\alpha = -75^o$and $\beta = -39^o$ is the optimum combination of winding angle. This combination not only sustains the loading conditions, but also maximizes the angle of twist.

V. APPENDIX

```
q=3e6                                         %internal pressure
P=-25e3                                       %Axial compression
R=0.0255                                      %Thickness of the prepreg
L=.300                                        %tube length
E1=236e9                                      %Young's modulus in fibre direction
E2=5e9                                        %Young's modulus in transverse direction
G12=2.6e9                                     %longitudinal shear modulus
v12=0.25                                      %longitudinal Poisson's ratio
Xt=38e8                                       %Maximum tensile strength in fibre direction
Xc=686e6                                      %Maximum compression strength in fibre direction
Yt=41e6                                       %Maximum tensile strength in transverse direction
Yc=107e6                                      %Maximum compression strength in transverse direction
S=69e6                                        %Maximum shear strength
p=0
j=0
fori=-75:-30
b=-75
while (b>=-75 & b<=-30)
aa=i;                                         % Alpha
bb=b;                                         % Beta
Theta=[aa;bb]
t =[0.00025; 0.00025; 0.00025; 0.00025]
c=1-v12^2*E2/E1
Q=[E1/c v12*E2/c 0; v12*E2/c E2/c 0; 0 0 G12]
s=sind(Theta(1))
c=cosd(Theta(1))
T1=[c^2 s^2 -2*c*s; s^2 c^2 2*c*s; c*s -c*s c^2-s^2]          % Transformation matrix of alpha
s=sind(Theta(2))
c=cosd(Theta(2))
T2=[c^2 s^2 -2*c*s; s^2 c^2 2*c*s; c*s -c*s c^2-s^2]          % Transformation matrix of beta
t1=transpose(T1)                                               % Transpose matrix of T(a)
Q1=T1*Q*t1                                                          % Global reduced stiffness for
alpha
t2=transpose(T2)                                               % Transpose matrix of T(b)
Q2=T2*Q*t2                                                     % Global reduced stiffness for beta
A=2*Q1*0.00025+2*Q2*0.00025
B=0.00025^2*Q1+0.00025^2*Q2
N1=[0.5*q*R; q*R; 0]                                           % Nx, Ny, Nz for case 1
```

```
sx1=inv(A)*N1                     % strain in X,Y,Z system for case 1
s1=t1*sx1                         % strain in 1,2,3 system for lamina
1
st1=Q*s1                          % stress in 1,2,3 system for lamina
1
s2=t2*sx1                         % strain in 1,2,3 system for lamina
2
st2=Q*s2                          % stress in 1,2,3 system for lamina
2
N2=[P/(2*pi*R);0;0]               % Nx, Ny, Nz for case 2
sx2=inv(A)*N2                     % strain in X,Y,Z system for case 2
s3=t1*sx2                         % strain in 1,2,3 system for lamina 1
st3=Q*s3                          % stress in 1,2,3 system for lamina 1
s4=t2*sx2                         % strain in 1,2,3 system for lamina 2
st4=Q*s4                          % stress in 1,2,3 system for lamina 2
Twist=((sx2(3,1)*L)/R)*180/pi
Twistplot(1+p,:)=Twist
Angleplot(1+j,:)=aa
Angles(1+j,1)=aa
Angles(1+j,2)=bb
Angles(1+j,3)=Twist
j=j+1;
b=b+1;
p=p+1;
End
End
```

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